

Optical properties of graphene: the Fermi liquid approach

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Abstract. - Optical properties of two-dimensional massless Dirac fermions are considered by the formalism of pseudospin precession equations which provides an easy and natural semiphenomenological way to include correlation effects. It is shown that the latter are negligible, with the only assumption that the system under consideration is normal Fermi liquid. This result probably explains recent experimental data on the universal optical conductivity of graphene (Nair R. R. et al, *Science* **320** (2008) 1308).

The recent discovery of the first purely two-dimensional material, graphene [1, 2] and of its peculiar electronic spectrum with chiral massless charge carriers (“Dirac fermions”) [3, 4] lead to an explosion of scientific activity (for review, see Refs. [5–7]). Among other unique properties of graphene, its universal optical conductivity is of special interest. It was demonstrated experimentally that visual transparency of graphene is determined only by the fine structure constant [8] (the same universal optical conductivity has been observed for graphite [9]). This result is in agreement with the theory for noninteracting Dirac fermions (see Refs. [10, 11] and references therein), within an accuracy of 5%. However, this is a problem since, generally speaking, one could expect an essential many-body renormalization of the optical conductivity. It has been demonstrated before the discovery of graphene that Coulomb interactions modify drastically the properties of two-dimensional Dirac fermions making them a marginal Fermi liquid with strong logarithmic renormalization of the Fermi velocity and related properties [12]. Recent explicit calculations [13, 14] result in corrections to the frequency-dependent conductivity $\sigma(\omega)$ of order of $1/\ln(W/\hbar\omega)$ where W is a cutoff energy of the order of the bandwidth. These corrections are for sure larger than the experimental errors.

Actually, the relevance of correlation effects in graphene is still rather controversial. In particular, direct measurements of electron compressibility in graphene [15] do not find any essential difference with the predictions of the noninteracting Fermi-gas model. At the same time, some many-body features were observed in the infrared con-

ductivity [16]. Theoretically, the problem seems to be also rather complicated. For example, taking into account ripples on graphene and related gauge fields [17–19] can drastically change the picture of electron-electron interactions [13, 20].

In this situation it seems reasonable to investigate the problem semi-phenomenologically, in the spirit of the Landau Fermi liquid theory [21–23]. The applicability of this theory to charge carriers in graphene is unclear now. However, in the current controversial situation it may be reasonable to start “from the answer”. It will be shown here that, in the framework of the Fermi liquid theory, the correlation renormalization of the optical conductivity are almost cancelled so that the experimental results [8] may find a natural explanation. This is not trivial since, in a standard situation, the Fermi liquid theory allows to include all correlation effects in the renormalization of parameters only for *static* properties whereas at finite frequencies essentially many-body effects can be expected [22, 23]. Since no alternative explanations are known yet it may be a serious motivation for a deeper microscopic study which would allow to justify the Fermi liquid theory for graphene.

We will restrict ourselves to the model of Dirac fermions neglecting valley and spin degrees of freedom. Then, the effective Hamiltonian for noninteracting fermions in the presence of a uniform time-dependent electric field $\mathbf{E}(t)$ reads

$$H = \sum_{\mathbf{p}} \Psi_{\mathbf{p}}^{\dagger} (v\mathbf{p}\sigma - ie\mathbf{E}\nabla_{\mathbf{p}}) \Psi_{\mathbf{p}} \quad (1)$$

where v is the electron velocity, \mathbf{p} is the quasimomen-

tum, $\sigma = (\sigma_x, \sigma_y)$ are the Pauli matrices, e is the electron charge, and $\Psi_{\mathbf{p}}^\dagger = (\psi_{\mathbf{p}1}^\dagger, \psi_{\mathbf{p}2}^\dagger)$ are electron creation operators depending on the pseudospin (sublattice) index $i = 1, 2$; here and further $\hbar = 1$. The canonical transformation

$$\begin{aligned}\psi_{\mathbf{p}1} &= \frac{1}{\sqrt{2}} (\xi_{\mathbf{p}1} + \xi_{\mathbf{p}2}), \\ \psi_{\mathbf{p}2} &= \frac{\exp(i\phi_{\mathbf{p}})}{\sqrt{2}} (\xi_{\mathbf{p}1} - \xi_{\mathbf{p}2})\end{aligned}\quad (2)$$

introduces the annihilation operators for the hole and electron states, $\xi_{\mathbf{p}1}, \xi_{\mathbf{p}2}$ with the energies $\varepsilon_{\mathbf{p}1,2} = \mp v p$, respectively, $\phi_{\mathbf{p}}$ is the polar angle of the vector \mathbf{p} (for a detailed discussion of the Hamiltonian and transformation see, e.g., Ref. [24]). In the collisionless limit the equation of motion for the average density matrix $\rho_{\mathbf{p}} = \Psi_{\mathbf{p}}^\dagger \Psi_{\mathbf{p}}$ has the form

$$i \frac{\partial \langle \rho_{\mathbf{p}} \rangle}{\partial t} = \langle [H, \rho_{\mathbf{p}}] \rangle = v \mathbf{p} \cdot \langle [\sigma, \rho_{\mathbf{p}}] \rangle - i e \mathbf{E}(t) \cdot \nabla_{\mathbf{p}} \langle \rho_{\mathbf{p}} \rangle. \quad (3)$$

Introducing scalar (n) and pseudospin (\mathbf{m}) densities by a decomposition

$$\langle \rho_{\mathbf{p}} \rangle = n_{\mathbf{p}} \mathbf{I} + \mathbf{m}_{\mathbf{p}} \sigma \quad (4)$$

where \mathbf{I} is the two by two unit matrix, one has a set of uncoupled equations of motion,

$$i \frac{\partial n_{\mathbf{p}}}{\partial t} = -i e (\mathbf{E} \cdot \nabla_{\mathbf{p}}) n_{\mathbf{p}}, \quad (5)$$

$$i \frac{\partial \mathbf{m}_{\mathbf{p}}}{\partial t} = 2i v (\mathbf{p} \times \mathbf{m}_{\mathbf{p}}) - i e (\mathbf{E} \cdot \nabla_{\mathbf{p}}) \mathbf{m}_{\mathbf{p}}. \quad (6)$$

Only the second one is relevant for us since the electron current does not depend on $n_{\mathbf{p}}$:

$$\mathbf{j} = 2e v \sum_{\mathbf{p}} \mathbf{m}_{\mathbf{p}}. \quad (7)$$

Further we will consider only linear optical effects assuming $\mathbf{E}(t) = \mathbf{E} \exp(-i\omega t)$ and using the linear approximation for $\mathbf{m}_{\mathbf{p}} = \mathbf{m}_{\mathbf{p}}^{(0)} + \delta \mathbf{m}_{\mathbf{p}} \exp(-i\omega t)$, $\delta \mathbf{m}_{\mathbf{p}} \sim \mathbf{E}$.

At last, using the unitary transformation (2), we derive

$$\mathbf{m}_{\mathbf{p}}^{(0)} = \frac{\mathbf{p}}{2p} (f_{\mathbf{p}1} - f_{\mathbf{p}2}) \quad (8)$$

where $f_{\mathbf{p}i} = \langle \xi_{\mathbf{p}i}^\dagger \xi_{\mathbf{p}i} \rangle$ are the Fermi functions of the energies $\mp v p$. This vector lies in the xy plane and therefore δm^z is not coupled with the electric field. Excluding δm^z from Eq.(6) (further we will omit the subscript \mathbf{p} for brevity) we obtain

$$\begin{aligned}(\omega^2 - 4v^2 p_y^2) \delta m^x + 4v^2 p_x p_y \delta m^y &= -i e \omega E \frac{\partial m^{x(0)}}{\partial p_x}, \\ 4v^2 p_x p_y \delta m^x + (\omega^2 - 4v^2 p_y^2) \delta m^y &= -i e \omega E \frac{\partial m^{y(0)}}{\partial p_x}\end{aligned} \quad (9)$$

where we have chosen the direction of x axis along the electric field. After a straightforward transformations we find $j_x = \sigma(\omega) E$ where the proportionality coefficient, that is, the optical conductivity, equals

$$\sigma(\omega) = -\frac{8ie^2 v^3}{\omega} \sum_{\mathbf{p}} \frac{p_y}{\omega^2 - 4v^2 p^2} \left(p_y \frac{\partial m^{x(0)}}{\partial p_x} - p_x \frac{\partial m^{y(0)}}{\partial p_x} \right). \quad (10)$$

For the case of zero doping and zero temperature, $f_{\mathbf{p}1} = 1, f_{\mathbf{p}2} = 0$ and we have a well-known result for universal, frequency-independent optical conductivity $\sigma(\omega) = e^2/16 = \pi e^2/8h$ (per valley per spin) [8–11].

This method of derivation can be easily generalized on the case of *interacting* electrons in the Fermi liquid theory approach. The only essential difference with the standard case [22, 23] is that we have to work with the matrix distribution function $\langle \rho_{\mathbf{p}} \rangle$.

The interaction effects are taken into account by the replacement $[H, \delta \rho_{\mathbf{p}}] \rightarrow [H, \delta \rho_{\mathbf{p}}] + [\delta H, \rho_{\mathbf{p}}^{(0)}]$ in the linearized version of the equation of motion (3) where

$$\delta H_{\mathbf{p}} = \sum_{\mathbf{p}'} F_{\mathbf{p}\mathbf{p}'} \langle \delta \rho_{\mathbf{p}'} \rangle \quad (11)$$

and $F_{\mathbf{p}\mathbf{p}'}$ is the (matrix) Landau interaction function [21–23]. For the case under consideration (zero doping and zero temperature) $\langle \rho_{\mathbf{p}}^{(0)} \rangle = -\frac{\mathbf{p}\sigma}{2p}$.

Now we have to use symmetry considerations to specify the Landau function. First, F should be rotationally invariant in the two-dimensional space. Second, due to inversion and time-reversal symmetry, it cannot contain σ^z matrices [25]. Third, it should vanish at $\mathbf{p}' \rightarrow 0$ or $\mathbf{p} \rightarrow 0$ since electron-electron interactions cannot open the gap without symmetry breaking [25]. Thus, we have

$$F_{\mathbf{p}\mathbf{p}'} = A \mathbf{I} \otimes \mathbf{I}' + B (\mathbf{p}\sigma) \otimes (\mathbf{p}'\sigma') + C (\mathbf{p}\mathbf{p}') (\sigma^x \otimes \sigma'^x + \sigma^y \otimes \sigma'^y) \quad (12)$$

where A , B , and C are some unknown functions of $|\mathbf{p} - \mathbf{p}'|$. In particular, the long-range Coulomb (Hartree) interaction singular at small momentum transfer contributes to the A function only. As for the “exchange” interactions B and C one can assume that they are smooth functions which can be expanded in powers of the momentum transfer square. Substituting Eqs.(11) and (12) into the equation of motion one finds:

$$\begin{aligned}\omega^2 \delta m^x - 4v^2 p_y^2 \delta \tilde{m}^x + 4v^2 p_x p_y \delta \tilde{m}^y &= -i e \omega E \frac{\partial m^{x(0)}}{\partial p_x}, \\ 4v^2 p_x p_y \delta \tilde{m}^x + \omega^2 \delta m^y - 4v^2 p_y^2 \delta \tilde{m}^y &= -i e \omega E \frac{\partial m^{y(0)}}{\partial p_x}\end{aligned} \quad (13)$$

where $\delta \tilde{\mathbf{m}} = \delta \mathbf{m} + \Delta$,

$$\Delta_{\mathbf{p}} = \frac{1}{v p} \sum_{\mathbf{p}'} [B_{\mathbf{p}\mathbf{p}'} \mathbf{p} (\mathbf{p}' \delta \mathbf{m}_{\mathbf{p}'}) + C_{\mathbf{p}\mathbf{p}'} (\mathbf{p}\mathbf{p}') \delta \mathbf{m}_{\mathbf{p}'}] \quad (14)$$

contains all correlation effects. Eq.(13) is the analog of Eq.(9), differing by the terms with Δ . The latter give an additional contribution to the current density which can be represented, after simple transformations, in the form

$$j_x^{corr} = 8e^2 v^3 \sum_{\mathbf{p}} \frac{p_y}{\omega^2 - 4v^2 p^2} (p_y \Delta^x - p_x \Delta^y) \quad (15)$$

Note that the terms with B function are exactly cancelled in the expression (15) and only the term proportional to C can, in general, survive. However, it vanishes obviously by symmetry if one assume $C = const$.

To find the correlation corrections to the optical conductivity explicitly in a general case one has to solve the integral equations (13),(14). Fortunately, their frequency dependence can be found just analyzing perturbation expansion in the interaction functions B and C . One can see (it is also obvious from physical considerations) that any adsorption processes require at least one real (and not virtual) interband transition and, thus, either p or p' should be equal to $\omega/2v$ (imaginary part of the fraction in Eq.(15) contains the delta-function). The leading correlation terms just vanish as was discussed above.

The next terms of the expansion of C in $(\mathbf{p} - \mathbf{p}')^2$ should be taken into account which gives, at least, one more power of p in the integrand. As a result, $j_x^{corr} \propto \omega^3$ which means, in dimensionless units, $(\hbar\omega/W)^3$. Indeed, since the interaction constant in graphene $e^2/\hbar v$ is of order unity, the energy cutoff is the only relevant characteristic which enters the problem. This is smaller than the corrections to the optical conductivity because of inaccuracy of the Dirac Hamiltonian itself [11], which are of order of $(\hbar\omega/W)^2$. Thus, the Fermi liquid interaction contributions to $\sigma(\omega)$ are really negligible.

To conclude, experimental data [8], together with the present analysis, seem to support the Fermi liquid picture of charge carriers in graphene, against the marginal Fermi liquid. The latter, according to the calculations [13] predicts many-body renormalization of the optical conductivity of order of $2/\ln(W/\hbar\omega)$, that is, of order of unity. At the same time, our consideration is purely phenomenological and microscopic justification of the Fermi liquid picture for graphene is required.

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